ELEMENTARY GRAPHICAL ARCHAEOMETRY

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I. Some Uses of Graphs

Although graphical techniques are employed in some of the more quantitative aspects of prehistoric archaeometry, there are many occasions when the use of a graph might not only clarify a particular problem but may also disclose a solution. Most graphs are visual methods of showing how one set of measurements varies with another[1]. The measurements or parameters must be related to each other and the graphs resulting from the plotting of two associated parameters yield information that cannot be obtained from a study of each parameter separately. In geochronology, for example, it would be futile to attempt to consider the phases of the last ice age without reference to time. It is, in fact, almost impossible to think about climate variations without some form of graph to depict the temporal variations of temperature.

When interpreting any graph, it is important to realise that the data from which the graph has been produced are usually reliable facts based upon more or less accurate measurements. But the number of measured points on any graph is necessarily limited. Consequently, one of the important uses of graphs is the ability to interpolate between the points and to extrapolate beyond them. Obviously deductions based on extrapolations are less reliable than those based on interpolations, because the former are not guided by any experimental points beyond the actual limits of the graph. However, theoretical extrapolations of graphs in both directions can lead to the discovery of new facts, that may enable field archaeologists to save much time and labour by avoiding uninspired trial-and-error excavations at barren sites.

Where graphs cross either of the axes, the interpretation of the intercepts may yield additional information. For example, the intercept of the R graph in Fig. 1 indicates the upper depth limit, above which no rodents are to be found; this information happened

1 As distinct from the many useful forms of pictorial diagrams, such as a sectored circle of arbitrary radius or a series of columns of arbitrary width, which display by their areas or heights respectively, the relative magnitudes of a single or isolated set of numbers or measurements. (See also linear histograms on page 57).

to be of considerable value in explaining some of the unexpected results arising from this particular excavation. (See also on page 46 the interpretation of the intercept of the graph in Fig. 2 C.).

With archaeological and osteological measurements it is usually their variation through space or time that is of most interest. But, as has been stated above, graphs can be plotted between any two related parameters. For example, graphs yielding useful information could be plotted to show the variation in the amount of copper in bronze alloys of different densities, or the variation in population with changes in temperature, or the variation in the strength of fabrics with humidity, and so on. But such parameters as these are less common in archaeological investigations.

Another type of graph which can often be of use in archaeological work arises from the variation in numbers of items of different types; such, for example, as the numbers of flints of different geometrical forms, or the numbers of pollen grains from different plants, or the numbers of coins from different mints. Graphs based on numerical counts of this kind depend on statistical variations.

It is therefore convenient for the present purpose to divide graphs into the three categories: (1) Spacial variations, (2) temporal variations, and (3) statistical variations. A few examples of graphs illustrating these three categories and based on archaeological and osteological measurements from recent or current investigations will be considered in the following sections.

II. Spacial Variations

Measurements which vary with space may change along a line, over an area or throughout a volume. In other words the measurements may vary in one, two or three dimensions of space respectively.

(a) One dimensional space variations:

During a recent cave excavation in the Mendip Hills, Somerset, a Romano-British occupation layer overlaid what appeared to be a burial of pre-metal age human remains under a cairn of stones. There was nothing peculiar about this stratification until recent animal bones similar to those found in the Romano-British layer were found *below* the stone cairn. In the hope of explaining what seemed to be an anomalous situation, an ordinary Cartesian graph was plotted of the numbers of different types of remains (animal bones, human bones, pottery sherds, evidences of fire, etc.) against their respective depths — (the one-dimensional space parameter) — below the original surface of the cave floor. With pieces of pottery, lumps of charcoal, etc., which may vary very much in size, it is desirable to make allowances for very large or very small pieces. To avoid approximations of this kind, it is obviously best, when practicable, to plot total weights of each type of article against depths. The resulting graphs are shown in Fig. 1.

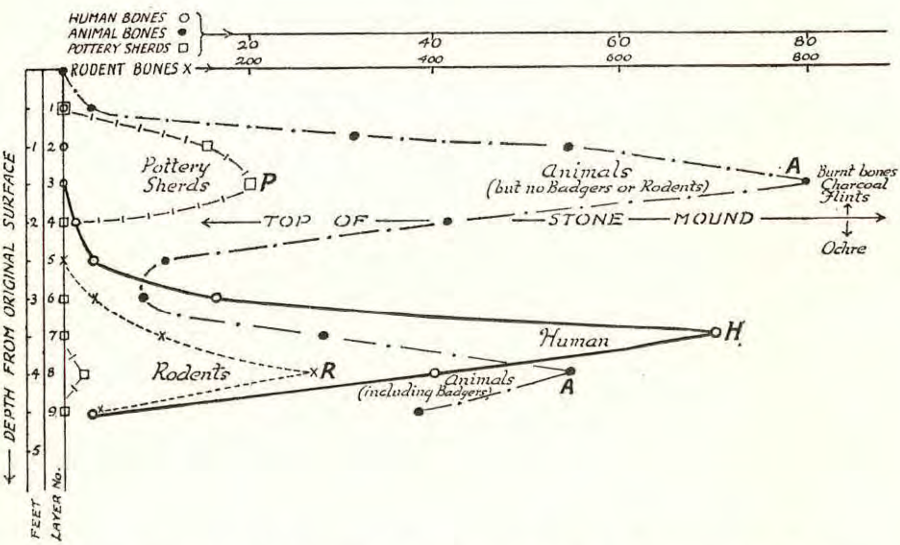

FIG. 1. VARIATION OF NUMBERS OF BONES AND SHERDS OF POTTERY WITH DEPTH BELOW THE FLOOR OF A CAVE IN THE MENDIP HILLS.

Several interesting facts are at once disclosed. Firstly, the Romano-British occupation layer is clearly depicted by the marked concentration of recent animal bones (curve A) at a depth of 1′ 6″. This peak coincides with the concentration of pottery sherds (curve P). Secondly, the lower concentration of recent animal bones is just below the level of the human bones. With the exception of two small sherds, no pottery and no charcoal was found at this lower level, but all the burrowing rodents (curve R) and all the badger remains were at this depth.

Because of the relatively large numbers of rodent bones, the abscissa scale units have been reduced to one-tenth of the value used for the other three graphs.

These four graphs not only disclose the precise nature of the distribution of the relics with depth, but the realisation of the presence of all the burrowing animals at the lower level offers an obvious explanation for the occurrence of the recent bones *below* human remains of an earlier date.

This excavation is referred to again when two-dimensional spacial variations are discussed.

Another example of a one-dimensional spacial variation arose in an examination of the laminae or strata in Roman "pigs" of lead. The laminated structure had been noticed many years ago by Professor W. Gowland[2] who mentioned that the layers were of tolerable uniformity. A closer examination confirmed by the graph A in Fig. 2 shows that the layers get gradually thinner towards the base of the pig. It is also obvious from graph B of Fig. 2 that they have a greater superficial area towards the base, the pig being in the form of a truncated tetrahedral. Because of these conclusions, the abscissae of these two graphs were multiplied together with the result shown in graph C. From this graph it is apparent that the laminae are of approximately equal *volume* at all depths greater than about one inch. The graph deviates towards larger volumes at the top of the pig where abnormally thick laminae occur. This arises because of the sunken panel containing a cast inscription on the top face. Much of the measured volume of the top lamina is, in fact, the air space occupied by the sunken panel.

The value of the maximum intercept on the abscissa axis, namely about 20 cubic inches, gives the minimum volume of the ladle used in filling the mould. It is also interesting to note that the leadsmith under-filled his ladle towards the end of the operation. This is revealed by the slight slope of the graph towards smaller volumes for the laminae at the base of the pig, which was, of course, the top of the mould when it was being filled.

(b) Two dimensional space variations:

In the case of variations in two dimensions or changes over a given area, several methods of depicting the changes are available.

[2] W. Gowland, 'The Early Metallurgy of Silver and Lead', *Archaeologia* LVII, pp. 359-422, 1901.

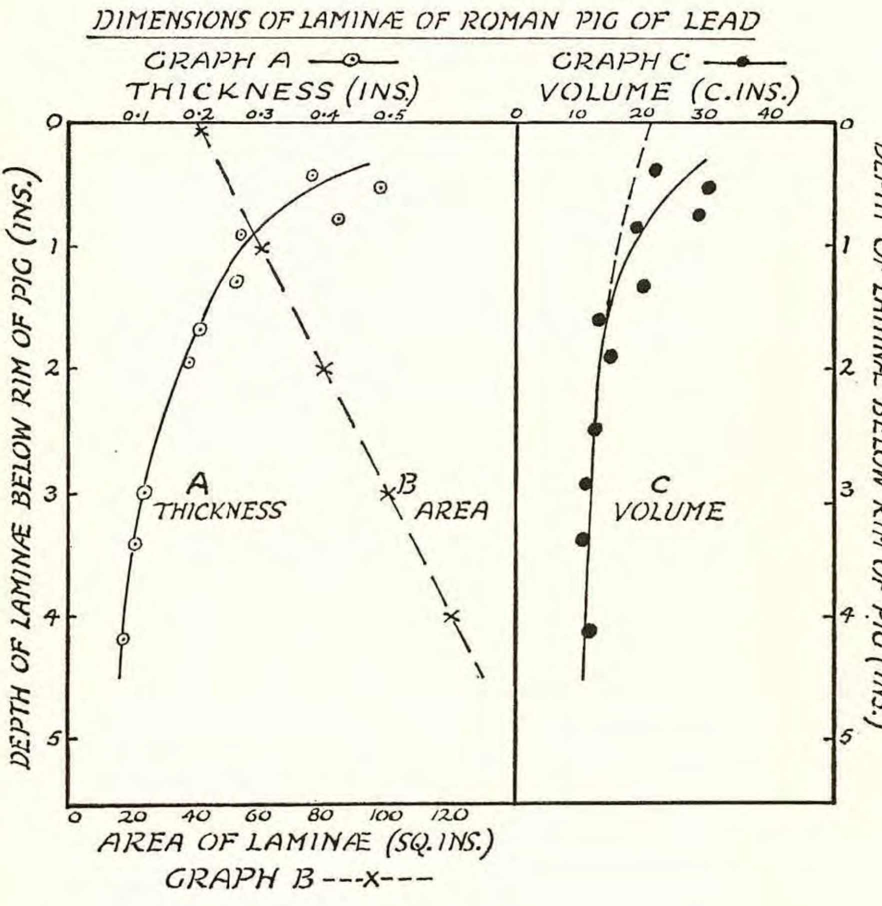

FIG. 2. VARIATION OF THE DIMENSIONS OF THE LAMINAE WITH DEPTH BELOW THE TOP OF A ROMAN PIG OF LEAD.

As a rule, the most useful method is to draw a reduced scale plan of the area concerned and to divide the area into a grid or series of small squares of given size, say one foot square. The numbers (or weights) of bones, artifacts or other materials, or the values of any measurements associated with each square at a given depth are written in the appropriate square on the plan. Lines are drawn connecting squares containing equal numbers or numbers which are equal to within small limits. These "equi-number" lines or isopsephic graphs may be looked upon as distribution "contours". It is convenient to call graphs of this kind "isographs" but "isopsephographs" is better from an etymological point of view. The word "contour" is somewhat misleading.

An alternative method of depicting two dimensional variations of the numbers or weights of relics of different kinds or of other measurements is to draw up a series or "family" of Cartesian graphs in which each graph depicts the change in concentration of the relics or of the particular measurements along a given line drawn on the surface (one of the grid lines, for example) and the "family" consists of the series of similar graphs along the parallel grid lines covering the whole area under investigation. Examples of these alternative methods of depicting two dimensional variations of archaeological data are shown in Figs. 3 and 4 respectively. They both refer to the cave excavation which provided the data for Fig. 1; but, when that cave was discussed, no mention was made of the two dimensional superficial distribution of the various finds in the Romano-British occupation layer or the distribution of the remains indicated by the lower peak of graph A. These are good examples for illustrating the value of isopsephographs. By plotting the numbers of different types of "finds" in each grid square and joining up the areas containing equal numbers within specified narrow limits, the three isopsephographs of Fig. 3 were obtained for the materials from the Romano-British layer situated 1 ft. 6 ins. below the surface. These graphs at once reveal the fact that food bones (I) and broken pottery (II) were concentrated between the fire-hearth (III) in front and a fairly vacant space to the rear, where presumably the occupants of the cave were wont to sit. They would be protected by the rock wall at their back and by the fire in front of their refuse heap. There may have been a small cooking fire at square 2 C. It would be difficult to disclose this situation in a more vivid manner.

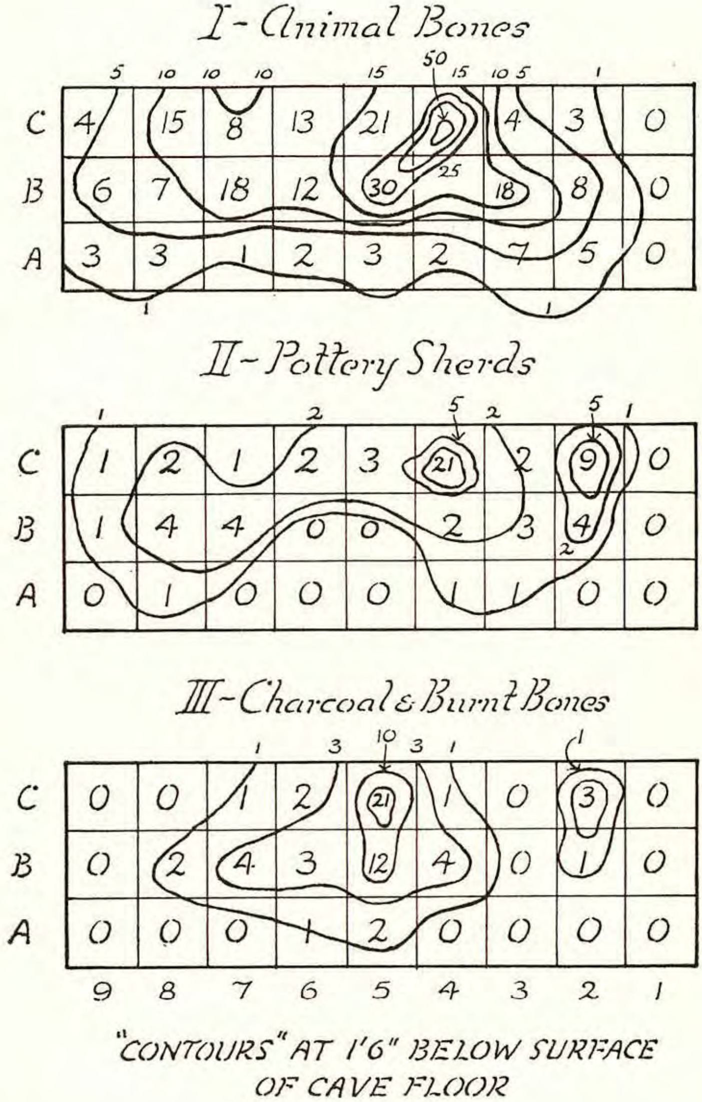

FIG. 3. ISOPSEPHOGRAPHS SHOWING THE DISTRIBUTION OF REMAINS FOUND 1′ 6″ BELOW THE FLOOR OF A CAVE IN THE MENDIP HILLS.

When an attempt was made to do the same thing for the animal bones of the lower peak of curve A (Fig. 1), it was found to be quite impossible to draw any kind of isopsephic graph. The bones were scattered randomly over the area of the cave in front of and beneath the cairn of stones. This is what would be expected with bones which had accumulated as the result of the work of badgers and burrowing rodents. This random distribution gives negative evidence as valuable as the positive results deduced from the isopsephographs of Fig. 3.

The same data from which these graphs were drawn have also been used to plot the three "families" of Cartesian graphs shown in Fig. 4. The same conclusions follow from these families of graphs, but it is felt that isographs are easier to understand and more realistic in their method of presentation.

Only when excavations have been carried out carefully and systematically can graphs be produced such as those shown in Figs. 3 and 4.

(c) Three-dimensional space variations:

As a two-dimensional isograph can be depicted as a family of one-dimensional Cartesian graphs, so a graph in three dimensions, or solid model may be shown as a family of two-dimensional isographs. Thus if isographs are drawn for the remains over the area of a cave at each successive foot, a 'solid' three-dimensional model results when the isographs are arranged horizontally one above the other at one foot intervals. The distribution of the relics in the spaces between successive layers would have to be interpolated from the distributions in the adjacent upper and lower isographs in the same way that interpolations are made between adjacent points of a linear Cartesian graph or between adjacent lines of a two-dimensional isograph.

III. TEMPORAL VARIATIONS

When the shapes of human bones or the forms of artifacts change with time, the resulting graphs depict the temporal variations of the morphology of the bone or artifact. Such graphs are therefore closely related to physical and cultural evolution respectively[3].

[3] (a) L.S. Palmer, 'Graphical Osteometry and Evolution', *Man*. LVIII No. 254, Dec., 1958.

(b) L. S. Palmer, 'Graphical Methods for depicting Cultural Development', *Man*. LIX No. 64, Mar., 1959.

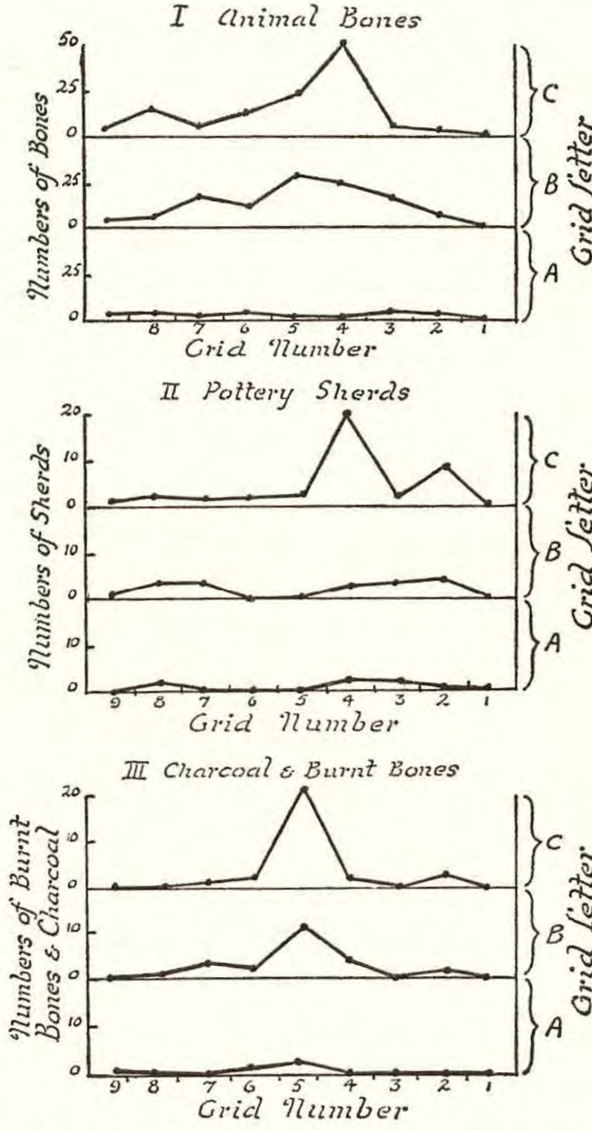

FIG. 4. FAMILIES OF CARTESIAN GRAPHS SHOWING THE DISTRIBUTION OF REMAINS FOUND 1′ 6″ BELOW THE FLOOR OF A CAVE IN THE MENDIP HILLS.

As with space variations, it is usual to plot the related parameters on a Cartesian system of axes. Furthermore, with graphs of this kind the times or chronological data are conveniently plotted as abscissae so that years B.C. increase to the left and years A.D. to the right of the zero BC/AD, which can be placed at any convenient point. Then the skeletal measurements or quantitative cultural criteria can be plotted as ordinates. As graphs of this kind have been fully described elsewhere[4], they need not be discussed in detail here. But some examples are given in Figs. 5, 6 and 7.

Fig. 5 is one of four graphs taken from *Man's Journey through Time*[5] and will serve to illustrate one method of depicting temporal variations of osteometric measurements. It should be emphasised that the linearity of this graph does not prove any genetic relationship between the several types of men. The graph refers merely to one aspect of man's morphology; namely, his erectness of posture, which is to some extent related to the height of the nuchal muscle area above the Frankfurt plane[6]. It is not necessary here to discuss fully the interpretation of this graph, nor to give details of the method by which the "percentage humanity" values of the ordinates are calculated from measurements of the nuchal area height index[7]. This graph will, however, serve to illustrate the importance of extrapolations. The backward extrapolation indicates that the graph would pass close to the comparable ordinate value for *Proconsul africanus*. From this it can be concluded that, at least as far as erectness of posture is concerned, it is not impossible for man to have descended (or ascended) from this Miocene ape.

The forward extrapolation of the lower branch leads to the point A; the comparable ordinate value for the Australian aborigine. From this it may be surmised that the gerontomorphic Neanderthal men migrated south-eastwards from Europe sometime after 50,000 B.C. and survived more or less in isolation in the Australian continent. There are several other factors which support this hypothesis. On

[4] L. S. Palmer, *Man's Journey through Time* (Hutchinson, 1957).

[5] *Op. cit.* Fig. 53, Graph 1.

[6] W. E. Le Gros Clark, 'New Palaeontological Evidence bearing on the Evolution of the Hominoidea', *Quart. Journ. Geol. Soc.*, vol. 105, pp. 225-64, 1949.

[7] Palmer, 'Graphical Osteometry', *Man*. LVIII No. 254, App.; *idem* 'A Graphical Treatment of Temporal Changes in some Skeletal Measurements', *American Journal of Physical Anthropology*, vol. 17, No. 4, pp. 325-34, Dec., 1959.

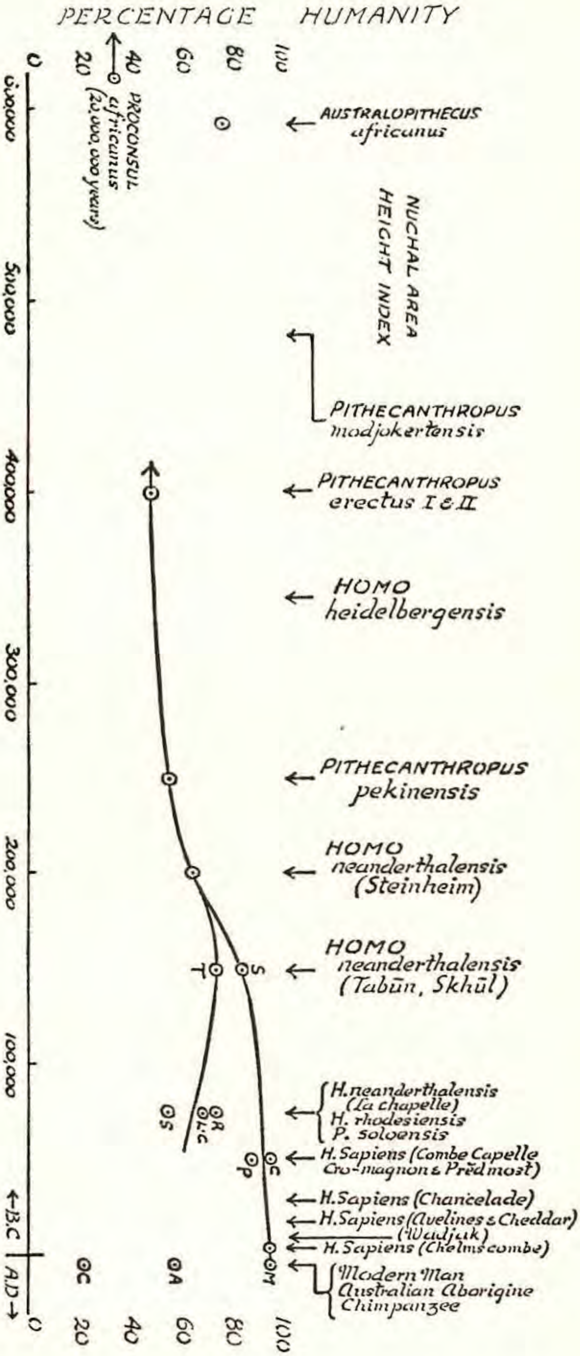

FIG. 5. VARIATION OF THE ERECTNESS OF POSTURE OF MAN AT DIFFERENT TIMES DURING HIS EVOLUTION.

the other hand, some factors, such as the facial characteristics[8] show no indication of any genetic relationship between these types of men. This suggests that further data are required before such a conclusion can be verified.

A graph which is relatively smooth and *regular* indicates some natural law connecting the two parameters upon which the graph is based. Thus the linearity of the earlier portion of the graph in Fig. 5 enables some estimate to be made of the natural law underlying the rate of development of man's erectness of posture. The gradient of the graph, which determines this rate, is about one and a quarter per cent per 10,000 years. This is a rate of one and a quarter "darwins"— the "darwin" being the unit of evolution defined by J. B. S. Haldane[9]. Any evolutionary rate derived from the slope of this graph necessarily assumes *some* genetic relationship between the Pithecanthropoids. A rate of evolution of just greater than one darwin accords closely with that obtained by Weidenreich when comparing the skull of *Pithecanthropus pekinensis* with that of *Homo sapiens*.

It would be very much more difficult if not impossible to come to the above conclusions concerning human evolution from a study of the table of skeletal measurements and dates from which this graph was produced.

Another example of the variation of a measurable parameter with time is shown in Fig. 6, in which the ordinates are the sum of the number of materials used, and the number of occupations undertaken by man at different times throughout his evolution during the past half-million years. It is suggested that these ordinate values are some measure of man's cultural development. The resulting graph, when plotted in the usual way, shows clearly the accelerated rate of cultural development which is known to characterise the diffusion of culture in Europe. The curve is not as smooth as this method of plotting seems to indicate. It is now fully recognised that cultural development has taken place in steps, due to the marked changes in man's social and economic life which followed the introduction of

[8] G. M. Morant, 'Studies in Palaeolithic Man II', *Annals of Eugenics II*, pp. 318-381, 1927.

[9] J. B. S. Haldane, 'Suggestions as to quantitative measurements of Rates of Evolution', *Evolution III*, p. 51, 1949.

the domestication of animals and the cultivation of the ground followed by the development of transport and the extension of trade and industry. The former occurred about 5,000 years B.C. and the latter about A.D. 1,000 It is obvious that cultural development at these times was so rapid that increments in the rate would not be distinguishable on the steeply ascending graph of Fig. 6. But if the

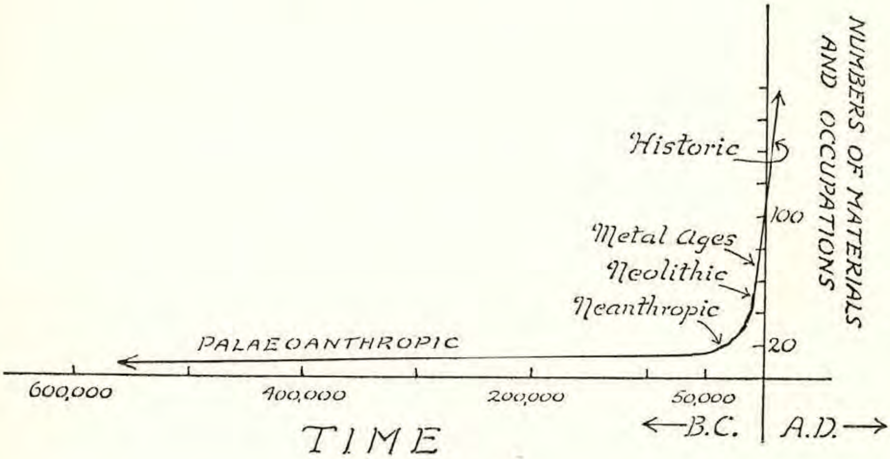

FIG. 6. VARIATION OF THE NUMBERS OF MATERIALS AND OCCUPATIONS ASSOCIATED WITH MAN AT DIFFERENT TIMES DURING HIS CULTURAL DEVELOPMENT.

ordinate values are reliable measurements of cultural status at any given time, the discontinuities discussed above must be inherent in the data. Dr. M. L. R. Pettersson[10] showed the obvious advantage of a log./log. plot[11] of the figures which would automatically exaggerate the changes in the cultural ordinate values for the more recent

[10] *Nature*, 184, p. 481, Aug. 8, 1959.

[11] Allometry involving the log./log. plot of biological growth data is not discussed here because it is considered to be outside the scope of a paper mainly concerned with archaeometry.

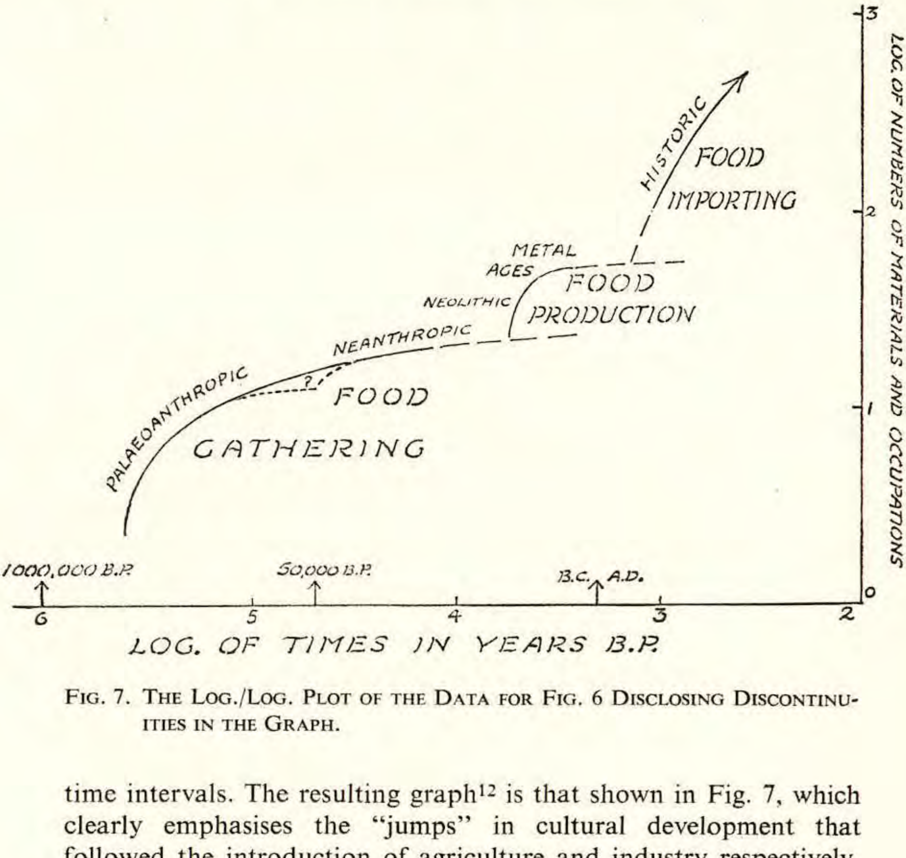

FIG. 7. THE LOG./LOG. PLOT OF THE DATA FOR FIG. 6 DISCLOSING DISCONTINUITIES IN THE GRAPH.

time intervals. The resulting graph[12] is that shown in Fig. 7, which clearly emphasises the "jumps" in cultural development that followed the introduction of agriculture and industry respectively.

As a final example of a graph with Time as one parameter, reference will be made to a comparatively recent and extremely interesting application of graphical techniques to an archaeological problem that has been made by Richard LeBaron Jr., an American engineer/archaeologist[13]. He plotted, as ordinates, the percentage ratio of the distance between the mast and the bow compared with the overall length of early Egyptian ships, the abscissae covering the period from 3,000 B.C. to 1,000 B.C.

[12] L. S. Palmer, 'Graphical Methods for depicting Cultural Development', *Man*. LIX No. 64, Mar., 1959, Fig. 2.

[13] "Egypt's Earliest Sailing Ships", *Antiquity*, vol. XXXIV, pp. 117-131, June, 1960.

From his graph (*loc. cit.* Reference 13, Fig. 10) he was able to show (with certain assumptions) the 'evolution' of man's knowledge of the art of sailing based on the gradual improvement in the position of the mast in the ship. Furthermore, he was able to deduce from the graph that it was highly improbable that a socket hole in a certain ancient model boat of known date could have been used for the insertion of the mast as had at one time been suggested.

IV. Statistical Variations

As one example of a statistical variation, the hoard of 1496 Roman coins found at East Harptree[14] on the Mendip Hills in Somerset will be considered. The graph or histogram of Fig. 8 A is one simple method of depicting the distribution of the coins which had been minted for nine different Roman emperors during the 77 years between A.D. 306 and 383 The ordinate values of the vertical lines represent the number of coins minted for each emperor. The total *length* of these lines is equal to 1496. The position of each line on the abscissa axis is the mid-year of the reign of the particular emperor for whom the coins were struck. A diagram of this type is a linear histogram in which the abscissae values are a series of dates arranged in chronological order. This differs from a graph, because there is no mathematical relationship between the ordinate and abscissa values; that is, in the present example, there is no natural relationship between the number of coins minted and the dates of the emperors' reigns. Hence, histograms cannot be interpolated or extrapolated like continuous graphs. (See also Footnote 1).

An area-histogram is a less simple but a more useful form of graph than a linear-histogram. An example is shown in Fig. 8 B. Here the ordinate value of each area is the mean or average number of coins minted during the period indicated by the abscissae. It is assumed that the coins were uniformly distributed throughout the period. Hence the product of the ordinate and abscissa values indicates the total number of coins issued during the period, and the whole *area* of the histogram is therefore, in the example under consideration, equal to 1496 coins in 77 years. This happens to be a more complicated graph than most area-histograms because the reigns of several of the emperors concerned happen to overlap.

14 John Evans, 'On a hoard of Roman coins found at East Harptree, near Bristol'. *Numismatic Chronicle*, vol. 8, 3rd series, No. 29, pp. 22-46, 1888.

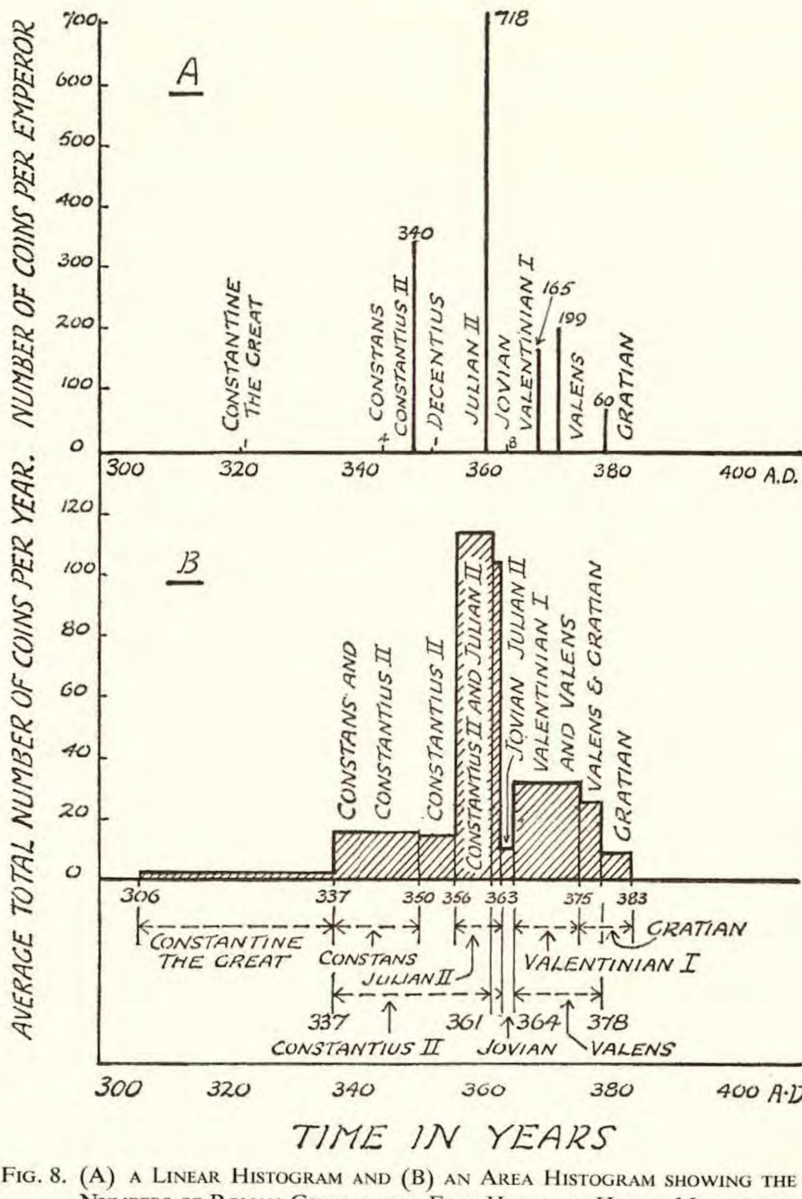

FIG. 8. (A) A LINEAR HISTOGRAM AND (B) AN AREA HISTOGRAM SHOWING THE NUMBERS OF ROMAN COINS IN THE EAST HARPTREE HOARD MINTED FOR DIFFERENT EMPERORS.

Coins could therefore have been minted for two or more emperors concurrently. Consequently, in such cases, the mean numbers of the coins for each of the emperors must be added together in order to obtain the total ordinate value for sections or areas during which more than one emperor reigned. The areas where this occurs are indicated below the abscissa axis and also on the area concerned.

From Fig. 8 B it is clear that the majority of the coins in this hoard were minted between the years A.D. 356 and 363 when both Constantius II and Julian II were in power. This conclusion cannot be deduced from the linear histogram of Fig. 8 A. The coins were probably buried during the reign of Gratian when the Romans were retreating from this country. This follows because no coins of later emperors occurred in the hoard.

As a final example of a statistical graph or histogram, reference will again be made to Roman pigs of lead. In the course of studying four Roman pigs of lead discovered near Green Ore on the Mendip Hills,[15] it was found to be necessary to determine the different numbers of Roman pigs in this country that had been made within small limits of given weights. The measurements were obtained for 48 pigs and the numbers of pigs, the weights of which approximated to the same value, were tabulated; the weights being recorded in both English pounds and Roman librae. The area of the resulting graph is 48 pigs of lead weighing a total of 7,470 pounds. From the tabulated values it was apparent that more pigs had been made to some weights than to others, but the histogram of Fig. 9 shows clearly that five times as many pigs weighed something in the neighbourhood of 195 *pounds* (or 270 librae) compared with the number that weighed about 195 *librae* (or 141 pounds). From other considerations[16] it was known that the Roman standard or specified pig should weigh 195 *librae*; but the histogram showed that the majority of pigs made in this country weighed 195 *pounds*. Apparently the British leadsmith conformed more or less to the specified number of 195, but used the British pound as his unit of weight. Roman pigs made in England thus tended to be too heavy. This conclusion, deduced from Fig. 9, provides a rational and quantitative explana-

15 L. S. Palmer and H. W. W. Ashworth, 'Four Roman Pigs of lead from the Mendips', *Proc. S.A.S.* vol. 101/2, pp. 52-88, 1956/7.

16 Palmer and Ashworth, *Proc. S.A.S.* vol. 101/2, pp. 72-3.

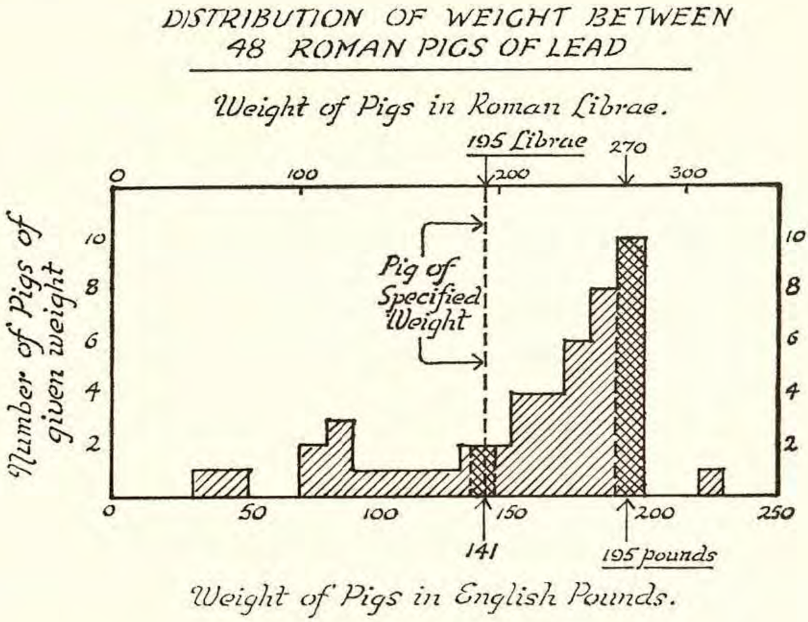

FIG. 9. AN AREA HISTOGRAM SHOWING THE NUMBERS OF ROMAN PIGS OF LEAD MADE TO GIVEN WEIGHTS IN ENGLAND.

tion for the peculiarly British practice of making overweight pigs. The excess weight was sometimes cold-stamped on the pig in Roman figures which, when subtracted from the weight of the pig expressed in librae, was always about 195 librae. Thus the histogram of Fig. 9 explains why numbers were cold-stamped on British pigs of lead and confirms their meaning as being the excess weight in librae above the specified weight laid down by the Roman authorities.

V. CONCLUSIONS

It will be appreciated that the types of graphs illustrating this paper are far from exhaustive. The particular examples chosen are intended to show some of the advantages of graphical treatments of archaeological and anthropological measurements. It will also be realised that at least two correlated parameters are necessary before graphical methods can be employed. This, of course, does not apply to the pictorial diagrams mentioned in Footnote 1.

Finally, it is felt to be advisable in this brief paper to limit the graphs to elementary examples and to avoid any mathematical equations and their differentials that may be employed for a more detailed analysis of linear graphs. Such considerations, although of possible academic interest, would not add appreciably to the archaeological information that can be obtained by employing the types of graphs and the methods of interpretation that have been outlined above.

VI. Acknowledgments

Of the nine Figures in the foregoing text, Numbers 2 and 9 are based on the Roman pigs of lead previously described in these *Proceedings* (Reference 15). Figure 5 has been re-drawn from the book mentioned in Reference 4. Figures 6 and 7 have been reproduced with permission of the Royal Anthropological Institute of Great Britain from the papers mentioned in Reference 3. Figures 8 A and B have been plotted from data recorded in Reference 14. Figures 1, 3 and 4 have been produced from analyses of materials that have been obtained from a Mendip Cave now being excavated by the Axbridge Caving Group and Archaeological Society under the supervision of the Author and Mr. J. Chapman. I am indebted to Mr. Chapman and the Society for permission to use information concerning this excavation, the results of which have not yet been published.

I am also pleased to have this opportunity of thanking my brother, Mr. E. B. Palmer, for drawing all the figures in the text, and of expressing my gratitude for a grant from the Muirhead Fund covering the cost of the blocks.